1 . Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

(3)

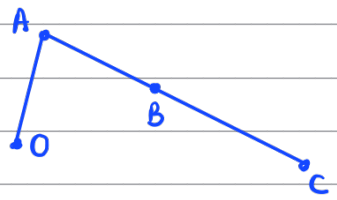
The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $|\overrightarrow{OD}|$, writing your answer as a fully simplified surd.

(3)

a) $A = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix}$, $C = \begin{pmatrix} -16 \\ p \\ 10 \end{pmatrix}$

if A, B and C lie on a straight line, then $\lambda \overrightarrow{BA} = \overrightarrow{CA}$



$\overrightarrow{BA} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \\ -1 \end{pmatrix}$ (1)

$\overrightarrow{CA} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} -16 \\ p \\ 10 \end{pmatrix} = \begin{pmatrix} 20 \\ -3-p \\ -5 \end{pmatrix}$

$\therefore \lambda \begin{pmatrix} 4 \\ -7 \\ -1 \end{pmatrix} = \begin{pmatrix} 20 \\ -3-p \\ -5 \end{pmatrix}$

$\underline{k}$: $-\lambda = -5 \quad \therefore \lambda = 5$

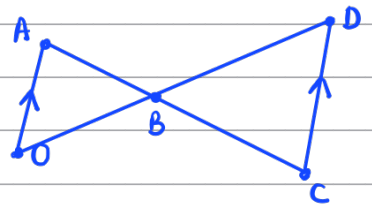
$\underline{j}$: $-7\lambda = -3 - p$

$-7(5) = -3 - p$ (1)

$\therefore p = 32$ (1)

$\therefore c = \begin{pmatrix} -16 \\ 32 \\ 10 \end{pmatrix}$

b)



OBD is a straight line

$\therefore \overrightarrow{OD} = \mu \overrightarrow{OB} = \mu \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix}$ (1)

if $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$, then $\overrightarrow{CD} = t\overrightarrow{OA}$

from diagram: $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$

$$\overrightarrow{OD} = \begin{pmatrix} -16 \\ 32 \\ 10 \end{pmatrix} + t \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$$

$$\therefore \mu \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} -16 \\ 32 \\ 10 \end{pmatrix} + t \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$$ (1)

$\underset{\sim}{j}$: $4\mu = 32 - 3t$

$4\mu + 3t = 32$ — ①

$\underset{\sim}{k}$: $6\mu = 10 + 5t$

$6\mu - 5t = 10$ — ②

$\therefore \mu = 5$, $t = 4$

$\therefore \overrightarrow{OD} = 5\begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 30 \end{pmatrix}$

$\therefore |\overrightarrow{OD}| = \sqrt{20^2 + 30^2}$ ①

$= 10\sqrt{13}$